COMMERCIAL BANK LIQUIDITY MANAGEMENT

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Bank liquidity management and optimal resource allocation of commercial bank Nostro accounts balances receive much less attention from the scientists compared to the questions on capital structure, funding, credit risk analysis and stress testing. Optimal liquidity management is a way to lower bank costs and risks, which are going to increase over time, especially when money markets are dry of free funds. There are two sides of the issue to be analyzed. The optimal resource allocation and corresponding accounts balances optimization are the first points to be considered. Then we should look at the process as a problem of liquidity requirements. In this article we formulate the procedure of optimal resource allocation up to a one-year horizon. The first part of the research is a one-month optimization: efficiency function and the related constraints of the corresponding accounts. The next step deals with the liquidity requirements (N3 and NSFR) restrictions, and balance sheet aggregation, and its influence on the liquidity requirements. With this in mind we develop inequations to specify long-term liquidity deficit. Finally, we create a consolidated mathematic model of optimal liquidity management up to a one-year horizon. In addition, we look at an alternative of short-term funding with premium to market to smoothen non-planned funds outflow.

Introduction

The aim of all banks’ transactions is to maximize their profits from fund raising and resource allocation. Due to interest rates term structure and subsequent features of liabilities structure, assets’ term of maturity is longer than the liabilities one [10], [14]. This situation leads to “liquidity gaps”, that is bank’s balance states that the sum of claims exceeds the sum of external account balances (Nostro account or corresponding account) with a restriction on open currency position limit.

Nostro account is a bank account opened in a corresponding bank to provide interbank transactions (most of the time). When a bank’s client performs a transaction (payment) to a client with a checking account in another bank, the sum of payment is written off Nostro account balance of the bank-sender and enters Nostro account balance of the bank-recipient. Nostro accounts must be opened for every currency (RUR, USD, EUR etc.) a bank provides transactions for. For example, corresponding account in the Central Bank of Russia is a Nostro account in Russian rubles.

Balance of every Nostro account changes during the transaction time due to clients’ funds movement and bank transactions (tax payments, interbank transactions, etc.). When Nostro account balance reaches zero\(^1\), a bank cannot satisfy clients’ claims any longer. This situations leads to financial and reputational risks realisation. Thus, the bank should manage and monitor Nostro account balances. Typically, there are two ways to manage liquidity. The first is for the bank’s balance sheet to have less claims than available short-term funds all the time. The second is short-term borrowing at money markets. Clients' funds markets or interbank markets are the sources of these short-term funds. We should note here that the bank must be sure to have adequate Nostro account balance to satisfy all claims with a minimum cost of (interbank) borrowing. So, every day \(k\), on a chosen forecast horizon, bank should maximize its profit function:

\[
\sum_{k=1}^{K} M_k \cdot i_k \rightarrow \text{max},
\]

where \(k\) is any day of a chosen horizon \(K\), \(M_k\) is interbank borrowing volume on a day \(k\), \(i_k\) is interbank interest rate (“cost of resources”).

In most cases, shorter duration of the transaction means cheaper resources (lower interbank interest rates). So, the cheapest way of borrowing appears to be a one-day (“overnight”) transaction, but it increases the risk of liquidity loss in case of short-term interbank shocks. Thereby, the regulators (Bank of Russia, Basel III committee recommendations) set limits on an assets-liabilities ratio.

High volatility of Nostro balances should be taken into account when constructing a model of Nostro accounts management. Large number of clients and no information about their transactions make it extremely difficult to predict corresponding accounts balances on a long-term horizon, and it is more appropriate to forecast

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\(^1\) Including intraday overdraft limit volume.
balance sheet aggregations instead. But, on the other hand, balance sheet forecast gives no information about external accounts balances. For example, decreased volume of liabilities can be distributed among bank’s own accounts, and increased volume of assets doesn’t lead to immediate with draws on a chosen day – they can be used later as a source of refinancing or distributed intrabank.

Thus, modern approaches (e.g. [3], [6], [9]) treating “liquidity deficit” as a balance sheet clearing are of little use for efficient management of bank liquidity. We need to create a method of liquidity management that is good (able to give recommendation in term of bank costs of optimal parameters of interbank transactions – currency, volume, duration) on both short and long-term horizons.

So, we introduce a new approach to the issue of liquidity management costs. In the proposed model a bank analyzes Nostro accounts balances changes and changes of the environment on every day basis and makes decisions about costs-optimal source, term and currency of the instrument of Nostro accounts management. We are going to use two approaches to determine optimal values of controlled variables on short and long-term horizons. On the long-term horizon we are going to determine them on the basis of deficit on liquidity requirements, and then we are going to smooth short-term fluctuations on the basis of Nostro accounts balances changes analysis.

The main prerequisites of the model are the following ones. Bank operates in the non-crisis environment, the resource demand can be fully satisfied from the interbank/intrabank [8] market (IM), and the supply can be fully allocated in the IM, the information about interest rates is available, and its value linearly depends on the term of the transaction only. Preterm claims on the interbank credits are not allowed.

So, the objective of liquidity manager (decision maker (DM)) is to minimize the sum of non-performing assets by determining the optimal term and volume of interbank transactions on a finite horizon with liquidity constraints [2]. Then, the DM should determine maximum deposit rate in case of clients’ demand, or optimal term, currency, and sum in case of IM.

**Liquidity management task on a short-term horizon**

Let’s have a look on liquidity management on a short-term horizon. We assume that DM is aware of balance clearance of payments $S_t$ in a given currency on a day $t$: $t = 1, … , T$, where $T$ is a finite horizon. The functional formula of interest rate on a term of the transaction is also known. $r$ is an interest rate on the positive (and negative) Nostro account balance. Interbank credit and deposit increases or decreases Nostro account balance respectively. So, the optimization function is [1], [3]:

$$(S_t - M_t) \times r + M_t \times i_M(d_k) \rightarrow \max, \text{ (2)}$$

where $S_t$ is balance of payments on a day $t$; $M_t$ is volume to interbank transaction $(+M_t - IM \text{ credit}, -M_t - IM \text{ deposit})$; $r$ is call deposit account interest rate; $i_M(d_k)$ is interest rate of the $M_t$ transaction and $d_k$ is a term of the transaction.

And the constraints are the following ones. The constraint on an open currency position (OCP) limit (3) restricts a profit loss due to reassessment of non-national currencies, the volume of limit is provided by the Regulator (Bank of Russia). The overdraft limit on a corresponding (Nostro) account (4) is the second constraint. If it is available, then $\text{Limit}^{corr}$ has negative value, if there is no overdraft limit, then $\text{Limit}^{corr}$ is zero. The third constraint takes into account urgent client payments of $t + 1$ day (5). Interbank market usually has shorter transaction time, than accounting system has, so we should have enough money to satisfy all clients’ demands before the market opening. The final two constraints are connected with liquidity constraints – instant liquidity ratio (6) and current liquidity ratio (7):

$$\text{sum}^c_{t=1} \sum_{k=1}^{m} (\text{CBR}^c - \text{Kurs}_k) \times S^\text{conv}_t \leq \text{Limit}^{opp}, \text{ (3)}$$

$$S_{t-1} + \text{In}_t - \text{Out}_t - M_t \geq \text{Limit}^{corr}, \text{ (4)}$$

$$S_{t+1} + \text{Limit}^{corr} > \text{Out}_t, \text{ (5)}$$

where $\text{In}_t, \text{Out}_t$ are outflow and inflow on a day $t$ respectively; $\text{Limit}^{corr}$ is Nostro account over draft limit; $\text{Limit}^{opp}$ is bank open currency position limit; $\text{CBR}^c$ is $S^\text{conv}_t$ is OCP summary.

Then the regulations of Bank of Russia add liquidity constraints [7]: instant liquidity ratio H2 (6) that regulates the risk of liquidity loss during a one-day period and current liquidity ratio H3 (7) that bounds liquidity risk on a one-month horizon.

$$\frac{S_t + A^{od}}{L_{od}} \geq \text{Limit}^{H2}, \text{ (6)}$$

$$\frac{S_t + A^{H3}}{L^{H3}} \geq \text{Limit}^{H3}, \text{ (7)}$$

where $A^{od}$ and $A^{H3}$ are numerators of bank liquidity constraints H2 and H3 respectively; $L^{od}$ and $L^{H3}$ are denominators of bank liquidity constraints H2 and H3 respectively.

Obviously, all decisions $M_t$ on a day affect future values of $S_t$, $t = 1 \ldots T$. So we can rewrite optimization function as:

$$\sum_{k=1}^{m} [M_t \times i_M^*(d_k) \times \frac{d_k}{K} + S^r_t \times \frac{r}{K}] \rightarrow \max. \text{ (8)}$$

And also we can expand (6) and (7) to estimate the impact of interbank transactions on the constraints. The numerator (Act) of (6) and (7) includes change of cash account balances in ATMs (NUs) and operational offices (NPodr) and cash resources in transit NCorr (including bank’s resources in cash centers of Bank of Russia), planned credit payments $l_i^j$ weighted by risk coefficient $w_f^j$ (varies for every credit category $j$), internal bank payments $Bint$, and interbank position $\delta M_t$. So:

$$\sum_{i=1}^{a} NU_{t-1, i} + C_{i}U_{t} + \sum_{j=1}^{b} NPodr_{t-1, j} + C_{i}Podr_{t} + \sum_{j=1}^{c} NCorr_{t-1, j} + + NC_{i}l - CI_{a}ut_{t} + Bint_{t} + \sum_{j=1}^{d} l_i^j \times w_f^j (l_i^j) \times \frac{d}{K} \times \frac{r}{K} \rightarrow \max. \text{ (9)}$$
The following notes should be taken into account. First of all, all cash account balances are taken on day $t-1$. Second, all balances in non-national currencies should be calculated in national coverage (multiplied by the official Bank of Russia exchange rate). Third, the value is restricted by (3) (restriction for OCP generated by currency conversion between Nostro accounts). Fourth, the mechanism of $\delta M$ can be described as follows: all interbank allocation deals with maturity values less than $t_{\text{norm}}$ are counted in, where $t_{\text{norm}}$ is a term of inclusion in liquidity constraint, so:

$$\delta M^*_t = \sum_{p=1}^{n} M^*_p. \quad (10)$$

(10) means that an interbank market deal volume is included into liquidity constraint calculation, when difference between contract term and days passed after the date of deal is less than the term of inclusion into liquidity constraint: $d_k - (t - 1) \leq t_{\text{norm}}$.

Similarly to (9) the denominator of liquidity constraints is formulated:

$$\sum_{m=1}^{k} \text{Depr}_{t-1} \times \text{PSch}_{t-1} + \text{POst}_{t-1} \times (c_j^f) + C_\text{LN}_{t} - C_\text{Out}_{t} - \sum_{j=1}^{n} [NU\text{S}_{s,j} + NU\text{Pod}_{d,j}] + \delta M^*_t. \quad (11)$$

where $\text{Depr}_{t-1}$, PSCh$_{t-1}$ is sum of clients’ deposits; POh$_{t-1}$ is opening balance of demand accounts (with a settling coefficient $c_j^f$ for every group $j$); $N\text{US}_{s,j} + N\text{Pod}_{d,j}$ are sum of cash demand of ATMs and operational offices on a day $t$; $\delta M^*_t$ is volume of interbank market deals of fund raising (calculated similarly to (10)).

**Liquidity management task on a long-term horizon and consolidated task**

In order to determine optimal values of long-term IM transactions we propose a method of liquidity scarce calculation according to liquidity ratios. Thus we can calculate sum of liquidity scarcity $A$ (13) by solving (11) and (12): $A^1 = \text{Limit}^H3 \times (\text{Pass}^H3 - cf \times \text{PStable}) - \text{Act}^H3, \quad (12)$

$A^2 = \text{Limit}^\text{NSFR} \times (\text{Pass}^\text{NSFR} - cf \times \text{PStable}) - \text{Act}^\text{NSFR}, \quad (13)$

$A = \max(A^1, A^2). \quad (14)$

where Pass$^H3$, Pass$^\text{NSFR}$ is sum of accounts balances included in denominator of chosen liquidity constraint (in our model it is H3 (Bank of Russia) and NSFR (Basel III)); Act$^H3$ and Act$^\text{NSFR}$ are sums of accounts balances included in numerator of chosen liquidity constraint; $cf \times \text{PStable}$ is stable part of demand accounts balances (calculation method is defined by liquidity constraint).

After the evaluation of $A$ we can formulate one more closely connected task: to determine optimal term of interbank transaction to minimize bank costs with the known liquidity scarcity $A$:

$$f(i(t), t) = A + i(t) \times \frac{A}{t} \rightarrow \min. \quad (15)$$

According to (10), after $t - t_{\text{norm}}$ moment an interbank deal will be included into liquidity constraint, and the DM will again face a challenge (15). This also means a double sum of $A$ in the $[t - t_{\text{norm}}, t]$ time interval that can be illustrated (exemplified by H3):

![Scheme of 2A generation on an $[t - t_{\text{norm}}, t]$ interval](image)

Double sum of $A$ leads us to increased costs, so, at first glance, minimum of (15) will be reached on $t = T + t_{\text{norm}}$. But at the second glance we see that the interbank market interest rate $i_t$ (with term $t < T$) is less than $i_r$, so, potentially, we have a minimum of (15) with term $t \leq T + t_{\text{norm}}$. So, the DM should solve a problem of optimal term $t_{\text{up}}$ of raising funds on an interval $(1; T + t_{\text{norm}}]$ with optimization function: $C(t_{\text{up}}) = \left[t_{\text{norm}} \times \left(\frac{T}{t_{\text{barr}} - t_{\text{norm}}}\right) \times A + \left(\frac{T}{t_{\text{barr}} - t_{\text{norm}}}\right) \times t_{\text{m}} \times A\right] \times i(t_{\text{up}}), \quad (16)$

where $\left(\frac{T}{t_{\text{barr}} - t_{\text{norm}}}\right) \times t_{\text{m}} \times A$ are bank costs on a $T$ horizon, $t_{\text{norm}} \times \left(\frac{T}{t_{\text{barr}} - t_{\text{norm}}}\right) \times A$ are costs of $2A$ periods $(t_{\text{barr}} - t_{\text{norm}})$, $t_{\text{m}}$ – period between deals, $t_{\text{barr}}$ is maturity of interbank deal.

The important thing is that the raised funds $A$ will increase bank short-term liquidity volume (balances on corresponding accounts) $S\_2$, decreasing short-term liquidity deficit and giving an opportunity of short-term allocation with interest rate $i(t_{\text{alloc}}) < i(t_{\text{barr}})$. By defining $t_{\text{m}} = t_{\text{barr}} - t_{\text{norm}}$ we get:

$$C(t_{\text{barr}}, t_{\text{alloc}}) = A \times \left[n(t_{\text{barr}}) \times t_{\text{norm}} + T\right] \times \left[i(t_{\text{barr}}) - i(t_{\text{alloc}})\right], \quad (17)$$

where $n(t_{\text{barr}}) = \frac{T}{t_{\text{barr}} - t_{\text{norm}}}$ is quantity of funding deals, $i(t_{\text{alloc}})$ are interbank rates.

Adding to the issue of short-term liquidity gaps, which can be covered by short-term raising, we get (7) formulated as follows:

$$\Sigma_{i=1}^{T} \left[M_{k} + d_{k} + A\right] \times i_k^G(d_k) \times \frac{d_k}{T} + \left[(1 - d_{k})A + S_{k}\right] \times \frac{1}{T}. \quad (18)$$

where $d_{k}$ is part of $A$ allocated to cover short-term gaps, $d_{k} \in [0;1]$.

Combining (17) and (18) we get consolidated task of liquidity management, where efficiency function is:

$$\Sigma_{i=1}^{T} \left[A(t) \times \left[n(t_{\text{barr}}) \times t_{\text{norm}} + T\right] \times i(t_{\text{up}}) + \sum_{i=1}^{T} \left[M_{k} + d_{k} + A\right] \times i_k^G(d_k) \times \frac{d_k}{T} + \left[(1 - d_{k})A + S_{k}\right] \times \frac{1}{T}\right] \rightarrow \max. \quad (19)$$

And constraints are as follows:
\[ S_k^c = S_{k-1}^c + \text{ln}_k^c - \text{Out}_k^c + \text{In}_k^c, k = 1..T, \] (20)

\[ \min (S_k^c) \geq \text{Limit}^{H^2} \cdot L^{ad} - \text{A}^{ad}, \] (21)

\[ \min (S_k^c) \geq \text{Limit}^{H^3} \cdot \text{Pass}^{H^3} - \text{Act}^{H^3}, \] (22)

\[ \min (S_k^c) \geq \text{Limit}^{NSFR} \cdot \text{Pass}^{NSFR} - \text{Act}^{NSFR}, \] (23)

The solution of the task gives an answer to optimal \( M_k \) in currency \( c \), with a term \( d_k \).

Consolidated task (3), (5), (18) – (23) allows DM to solve the problem of optimal resource distribution both as a problem of liquidity gaps and as a problem of costs minimization of liquidity ratios satisfaction simultaneously.

**Additional results**

Consolidated task (3), (5), (18) – (23) considers the issue of optimal resource allocation both as a problem of liquidity gaps and as a problem of cost of liquidity ratios minimization, but the following question is still open: is there any other way to decrease costs of liquidity constraints regulation? Obviously, an alternative to interbank market is a money market of clients’ resources with the main feature (as a tool of liquidity management) being no available information of interest rate and volume of the market. So, the DM should decide on the maximum interest rate of short-term fund raising at the clients’ market. This task may be solved by comparing costs of long-term and short-term fund raising.

Here we’d like to describe the most common task – borrowing in deposits with preterm recall (demand and similar short-term deposits). According to (9) and (11) this transaction will lead to equal increase in numerator and denominator in liquidity constraint, so, according to (12):

\[ \text{Limit}_{norm} = \frac{\Delta(t_{norm})}{L(t_{norm})} = \frac{\Delta(t_{norm}) + d(S_k)}{L(t_{norm}) + S_k}, \] (24)

where \( S_k \) is raised funds on clients market; \( d(S_k) \) is the part of \( S_k \), that should be allocated up to \( t_{norm} \) in order to satisfy liquidity constraints.

Then we can calculate our revenue and costs of this transaction. The revenue is:

\[ R_{cl}(t_b, d) = (1 - d) \cdot S_k \cdot t_b + i_d(t_b) + d \cdot S_k \cdot t_{norm} \cdot r_{norm} \] (25)

where \( R_{cl} \) is bank’s revenue; \( S_k \) is volume of funds raised; \( t_b \) is optimal term of resource allocation (\( t_b + t_{norm} = t_n \)); \( t_n \) is estimated term of maturity of \( S \) (as it is fixed in the deal).

And the bank’s costs are as follows:

\[ C_{cl}(t_a, t_b) = S_k \cdot (i \cdot o(S_k) + t_k + t_a \cdot r_a(t_a) + (t_n - t_k) \cdot r_{norm} (1))/T. \] (26)

Where \( C_{cl} \) is bank’s costs (within \( t_b \) period); \( i \) is interest rate of borrowing; \( o(S_k) = (1 + \text{legal reserve requirement fund ratio}) \); \( t_k \) is forecasted term of maturity of \( S \) (\( t_k < t_n \)); \( t_a \cdot r_a(t_a) \) are costs generated by interbank market deals after \( t_k \) to satisfy liquidity constraints; \( (t_n - t_k) \cdot r_{norm} (1) \) is cost of deposit ‘call’, or any other similar short-term fund raising instrument for the period \( (t_n - t_k) \cdot r_a \).

It follows from this that borrowing is preferable until \( R_{cl}(t_b, d) \geq C_{cl}(t_a, t_b) \) and so we can calculate the \( i_{max} \) as:

\[ i_{max} = \frac{1 - d \cdot S_k \cdot t_b + d \cdot t_{norm} \cdot r_{norm} \cdot t_{norm} - r_{norm} \cdot t_k - r_{norm} (1) - r_a \cdot t_a}{o(S_k) \cdot T}. \] (27)

\( i_{max} \) is maximum interest rate of client’s market resources borrowing. Fundraising on any \( i \) less than \( i_{max} \) is preferable due to lower costs compared to long-term fund raising in the interbank market in order to satisfy liquidity constraints.

Thus, the DM should always compare a given alternative (27) before interbank transactions. If there is no way to raise enough funds on client’s market, then applying (3), (5), (18) – (23) will give optimal decision on currency, volume and term of interbank transaction with bank costs minimization criteria (including costs on liquidity ratio regulations).

But, still, there is a problem of forecasting short-term balances (on daily basis) that should be solved prior calculating this task. We are planning to formulate and solve the task of forecasting Nostro account balances [9] and to apply this model to real data in a bank.

The consolidated task described in this paper will help in liquidity management of bank’s costs minimization.

**References**


The date of the manuscript receipt: 20.10.2014

К ВОПРОСУ ОБ УПРАВЛЕНИИ КРАТКОСРОЧНОЙ ЛИКВИДНОСТЬЮ КОММЕРЧЕСКОГО БАНКА

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Исследование деятельности банка, направленной на выполнение своих обязательств по предъявляемым требованиям и оптимальное распределение свободных ресурсов во времени получило куда меньшее освещение в современном литературе и научных работах по сравнению с вопросами о структуре капитала и источниках его формирования, его обеспеченности, анализу кредитных рисков и стресс-тестирования. Тем не менее управление ликвидностью – одна из наиболее значимых сторон банковской деятельности, особенно в условиях снижения ликвидности по банковской системе и следующим за этим процессом удорожания ресурсов. Для разрешения задачи оптимального перераспределения ресурсов необходимо рассмотреть ликвидность с двух сторон: с точки зрения непосредственно оптимизации (минимизации) суммы остатков средств на внешних счетах и с точки зрения выполнения нормативов ликвидности банка. В данной статье приводится методика управления ликвидностью коммерческого банка на горизонте до одного года включительно. Рассматривается вопрос о формировании задачи управления ликвидностью на горизонте в один месяц, о формировании целевой функции и ограничений, связанных с остатками на внешних счетах. Далее формируются ограничения для нормативов ликвидности Н3 и NSFR и определяется группировка статей баланса, определяется влияние кумулятивных агрегатов на нормативы и вырабатываются неравенства, позволяющие определить уровень дефицита по нормативам. Формируются оптимизационные задачи на горизонте до одного года. Дополнительно рассмотрен вопрос о регулировании ликвидности за счет привлечения клиентских средств под ставки с премией к рынку в инструменты с возможностью досрочного отзыва.

Ключевые слова: ликвидность банка, краткосрочное моделирование, нормативы ликвидности, минимизация издержек, привлечение средств.

Please cite this article in English as: