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MODELING ECONOMIC GROWTH OF DIFFERENT COUNTRIES BY MEANS OF PRODUCTION FUNCTIONS ON THE BASIS OF COMPARATIVE ANALYSIS OF DYNAMICS OF INTERACTION OF SOCIAL GROUPS

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Basing on the theory of endogenous growth, we provide a possible explanation of differences in the development of national economies of the world and their reaction to the global economic crises. Earlier an approach to modeling of the choice of technologies in various countries was proposed (Matveenko, 2007, 2010). Technological progress is modeled as a change of parameter α of the Cobb-Douglas production function $Y = AK^{\alpha}L^{1-\alpha}$ or of the CES production function $Y = A(\alpha K^{p} + (1-\alpha)L^{p})^{1/p}$. The parameter α has both technological and institutional meaning for the economy, in particular, for the Cobb-Douglas function it is the capital share. Recently, a big difference in capital share in different countries and in different times has been found empirically by several authors. The choice of the parameter naturally depends on the interests of the social groups: the workers and the capital owners; each group agrees to a change of the parameter α if it leads to income increase for this group. Thus, it is possible to identify the areas of coincidence and non-coincidence of interests of the social groups on the plane α-k, where k is the capital-to-labor ratio. Using the UNSTATS statistical data, we build the graphs of parametric dependencies α-k for the economies of the USA, Japan, Russia, China and Iran for the period of 2000 – 2010. The graphs clearly identify the sub-period of the crises by changes in the area of coincidence/non-coincidence of interests. A specific behavior of the integral characteristic for the area of non-coincidence of interests of the social groups in Russia and China is noted, which differs radically from the situation in the USA.

Key words: modeling, economic growth, production function, dynamics of interaction of social groups.

Preface

The production function illustrates a country's product output depending on the input factors (capital K and labor L, seen as social groups). We assume that economic agents make decisions based on "the rational expectations principle" [10, 11]. Being aware of the fact that this viewpoint is not the only one (see [9], [13]) we can constructively analyze social groups' interests. Based on the endogenous growth theory [3-5,15] an attempt to explain the differences in the development of certain economies has been made through the use of modified α-CES production functions. In this case the principle factor is taking into account interests of various social groups (in terms of production functions that would be shares of production functions owners) in various countries during the 2008-2010 recessions. This approach was suggested in the listed works [3-5] with the example of the Cobb-Douglas production function to describe the interactions of social groups under conditions of economic growth regardless of a country. Later on this method was applied to the analysis of developing countries' social groups interactions [6]. Social groups employees as well as capitalists agree to change the α parameter of their respective GDP shares if it results in profits increase. Thus, in the α-*k* parametric volume it became possible to construct the primary social groups' interests concurrence/divergence area. The α-*k* parametric dependencies graphs were completed for the economies of the US, Japan, Russia, China, and Iran in

2000 – 2010. The obtained results in evolution for every country identify the recession period through changes in the interests concurrence area.

1. Production functions and social groups' interests

The modified CES-type production function looks the following:

$$F(K,L)=[\alpha(A_K K)^p+(1-\alpha)(A_L L)^p]^{1/p}. \quad (1.1)$$

Here A_K, A_L >0 - factors efficiency coefficients, $0<\alpha<1$, $p\in(-\infty,0)\cup(0,1)$ - parameters. CES function in the particular case $A_K=A_L$ looks:

$$F(K,L)=A[\alpha K^p+(1-\alpha)L^p]^{1/p}, \quad (1.2)$$

where A – total efficiency of the factors the value and factors substitution elasticity σ are bound through equation:

$$p=1-\frac{1}{\sigma}. \quad (1.3)$$

The production function's parameters differ for the countries and the time period. They reflect the country's institutions. Let's consider a model where the α weight parameter can be subject to selection by the factors owners. In the selected production function they represent social groups of capitalists and employees.

We will define the factors owners' interests on condition of their profit increase, $(\partial F/\partial K)K$ and $(\partial F/\partial L)L$. The condition for the capitalists' income increase in case $A_K=A_L$ is the inequation

$$\frac{\alpha}{p}\left[1-(k)^p\right]<1, \quad (1.4)$$

where $k=\frac{K}{L}$. The condition for the employees' income increase is the inequation

$$\frac{(1-\alpha)}{p}\left[1-\left(\frac{1}{k}\right)^p\right]>1. \quad (1.5)$$

Let's specify two cases: 1) $0<k<1$,- labor is an excessive factor; 2) $k>1$-capital is an excessive factor. In the former case let's first address the interests of capitalists. In (1.4) the form in square brackets is negative, while the leftmost part is positive. The condition for the capitalists' income increase is equal to the inequation

$$\alpha<\frac{p}{1-(k)^p}. \quad (1.6)$$

Note, that the right-side function grows by *k* and reaches 1 at $k=a(p)=(1-p)^{1/p}$. That means that at $k\in(0,a(p))$ the capitalists are interested in increasing α only on condition (1.6), however, at $k\in(a(p),1)$ – unconditionally. As for the employees, in the former case the value in the in equation's (1.5) left half is negative, i.e. the inequation (1.5) is inconsistent. This means that, in this case, the employees are interested in decreasing the α parameter.

In the latter case the leftmost part of the inequation (1.4) is negative, i.e. solved by default: the capitalists are unconditionally interested in increasing the α parameter. Let's rewrite the (1.5) inequation into:

$$\alpha<1-\frac{p}{1-\left(\frac{1}{k}\right)^p}. \quad (1.7)$$

The rightmost part grows by *p* and changes its sign at point:

$$b(p)=\left(\frac{1}{1-p}\right)^{1/p}. \quad (1.8)$$

At $k\in(1,b(p))$ the employees are interested in lowering α, while at $k>b(p)$ and condition (1.7) – in increasing α. Thus, for the CES-function at $p<0$ on the (k,α) plane, there are two areas of social groups' interests concurrence:

a) Within the

$$\Omega_p^{\downarrow}=\left\{(k,\alpha): 0<k<a(p), \alpha>\frac{p}{1-(k)^p}\right\}$$ area, both social groups agree to decrease the α parameter;

b) Within the

$$\Omega_p^{\uparrow}=\left\{(k,\alpha): k>b(p), \alpha<1-\frac{p}{1-\left(\frac{1}{k}\right)^p}\right\}$$ area, both social groups agree to increase the α parameter.

Within the remaining area $0<\alpha<1$ the social groups' interests diverge.

2. Identification of production functions by statistical data

Application of the production function in the described model asks for defining and supplying of the required parameters and factors. Naturally, this can be done by using statistical data for the requested period. However, although the labor resources data are easy to obtain (e.g. United Nations Statistics Division) [16] there are no such data on capital. That is why we are going to use method [8], by which the model identification is performed through defining its external parameters based on historical data. A part of the parameters are evaluated directly on the basis of the available statistical data. The major part of the parameters is evaluated indirectly comparing the time series of the model's values against their statistical counterparts.

In the model let's take into account the country's foreign trade turnover as well as the changes of relative prices for the constituents of the basic macroeconomic balance. We'll do this by using certain defined functions, whose parameters we set by drawing on statistical data. Let $Y(t)$ be the uniform production

function for capital volumes $K(t)$ and labor $L(t)$ having constant substitution elasticity (CES-function)

The efficient cost of production assets changes according to the equation

$$\frac{dK}{dt} = J - \mu K, \quad K(0) = K_0 \quad , \quad (2.1)$$

where μ is the capital outflow rate, while $J(t)$ is the new capital increment rate (investments into the fixed capital).

Labor is quantified through the annual average number of people employed in the economy. Based on the statistical data we assume, that labor $L(t)$ grows at constant rate $\gamma > 0$ see [8],

$$\frac{dL}{dt} = \gamma L(t), \quad L(0) = L_0 \qquad (2.2)$$

Suppose that at any given moment t the basic macroeconomic product balance in current prices is drawn:

$$\rho_Y Y(t) + \rho_I I(t) = \rho_C C(t) + \rho_J J(t) + \rho_E E(t) \quad (2.3)$$

where the left sum is the sum of the GDP output and import $\rho_Y Y(t), \rho_I I(t)$, while the right sum is the total consumption by the populace, the government, and non-profit organizations with addition of net wealth accumulation along with the increment of material resources, $\rho_C C(t)$, investments into the fixed capital $\rho_J J(t)$, and export $\rho_E E(t)$. Through $\rho_Y, \rho_I, \rho_C, \rho_J, \rho_E$ the defined GDP deflators, import prices, ultimate consumption, investments, and export.

As we need the output values, those of investments, export, and import, expressed in fixed prices (2005 prices), let's address the product balance expressed in relative prices indices:

$$Y(t) + \pi_I(t) I(t) = \pi_C(t) C(t) + \pi_J(t) J(t) + \pi_E(t) E(t) \quad (2.4)$$

where the indices of import relative prices, investments and export are defined by the ratios:

$$\pi_Y(t) = \frac{\rho_Y}{\rho_Y} = 1, \quad \pi_I(t) = \frac{\rho_I}{\rho_Y}, \quad \pi_C(t) = \frac{\rho_C}{\rho_Y}, \quad \pi_J(t) = \frac{\rho_J}{\rho_Y}, \quad \pi_E(t) = \frac{\rho_E}{\rho_Y} \quad (2.5)$$

meanwhile the expression balancing equation (2.4), takes on the following form

$$Y(t) + \pi_I(t) I(t) = Q(t) + \pi_J(t) J(t) + \pi_E(t) E(t), \quad (2.6)$$

$$Q(t) = \frac{\rho_C}{\rho_Y} C(t)$$

To solve the aforementioned equation system it's necessary to define the investments volume $J(t)$, that of export $E(t)$ and import $I(t)$ at fixed 2005 prices, assuming these volumes are defined by fixed parameters.

The investments volume at fixed prices $J(t)$ is defined by the share ϑ of current investments cost within the sum of current import and export costs:

$$\vartheta = \frac{\pi_J(t) J(t)}{Y(t) + \pi_I(t) I(t)} \qquad (2.7)$$

The export volume at fixed prices $E(t)$ is defined by δ the export share in the output (at their current costs) :

$$\delta = \frac{\pi_E(t) E(t)}{Y(t)} \qquad (2.8)$$

The import volume at fixed prices $I(t)$ is defined by the import to GDP-export difference ratio ρ (at their current costs):

$$\rho = \frac{\pi_I(t) I(t)}{Y(t) - \pi_E(t) E(t)} \qquad (2.9)$$

To identify the model it's necessary to set the change of its external intensive parameters: relative prices: $\pi_I(t), \ \pi_C(t), \ \pi_J(t), \ \pi_E(t)$, fixed parameters: $\alpha, \ p, \ \gamma, \ \mu, \ \vartheta, \ \delta, \ \rho$ and three initial values: $K(0), \ L(0), \ Y(0)$, in order for the estimated macro-indices' time series (variable models) to be close to the statistical time series of a country's corresponding economic macro-indices. However, we'd like to point out that this is not the only way to evaluate the production functions' factors. Formulation of a marginal problem for the evaluation of a diversified economy's dynamic model is viewed in work [2].

3. Analysis of the social groups interaction dynamics

The social groups interaction dynamics can be evaluated by analyzing conditions, within a given period, reflecting those groups interests. Choosing the 2000-2010 period we expect the years of the financial-and-economic crisis to influence, in a certain way (indirectly through macroeconomic factors), the shape changes of the social groups' interests concurrence/divergence areas. Let's compare the social groups interaction dynamics in the USA, as an example of developed countries, against China, Russia and Iran as examples of developing countries. This choice is explained by the fact that economies of these countries are quite representative of general and particular world trends.

Currently, the USA is the country with the biggest economic potential and leading in economic development pace. The state and dynamics of the US economy set world economic development trends. The USA has been a leader in the world economy for 100 years, however, starting with 2000 due to recession and economic growth of developing countries, its influence went somewhat down.

Japan's economy is one of the most developed world economies. As for the volumes of GDP and industrial production output Japan is 3rd after the USA and PRC. At the same time it should be noted that for 15 years Japan's econmy has been in stagnation.

Today China plays an important role in the world economy.

Chinese economy's peculiarity is its dependence on the external market. China's export volume is ranked 1st in the world [1,12].

China and Russia are two rapidly developing BRICS members. Russia has the highest GDP per capita among BRICS members.

Iran has specific territorially isolated economy. Iran's economy is one of the richest in Asia, placed 18th in the world by its national product volume and largest among countries of Western Asia, Middle East, and OPEC. Iran's GDP is second largest in islamic world after Turkey. Not the least factor is the survival of Iran's economy under sanctions.

3.1 Analysis of the social groups interaction dynamics among developed countries (USA and Japan)

Let's analyze the US economy using the suggested model. Inequation (1.4) is the condition for capitalists' income growth, while in equation (1.6) is the condition for employees' income growth. We will use the UN statistical data [16].

Table 1

σ and *p* values for the US economy in 2000–2010

years	σ	*p*	*L(t)*	*K(t)*
2000	0.17907	-4.58456	136.891	2213.8
2001	0.16706	-4.9857	136.933	2189.8
2002	0.1618	-5.18047	136.485	2124
2003	0.16104	-5.20947	137.736	2191.7
2004	0.16795	-4.95432	139.252	2328.2
2005	0.17147	-4.83194	141.730	2452.5
2006	0.17248	-4.79776	144.427	2509.9
2007	0.16385	-5.10303	146.047	2470.8
2008	0.14864	-5.72754	145.362	2327.9
2009	0.12348	-7.09848	139.877	1956.4
2010	0.13067	-6.65305	139.064	1990.9

Case 1. Labor is an excessive factor. In this case $\alpha < -4{,}58458$. The rightmost function (1.6) grows by *k* and reaches value 1 at $k = a(p) = (1-p)^{1/p} = 0{,}68717$. This means that at $k \in (0;0{,}68717)$ capitalists are interested in growth α only given $\alpha < -4{,}58458$, while at $k \in (0{,}68717;1)$ – unconditionally. As for the employees, given $p < 0$, the inequation (1.7) is inconsistent. This means that, in this case, the employees are interested in decreasing the α parameter.

Case 2. Capital is an excessive factor. As $p < 0$ the leftmost part of the in equation is negative, the inequation (1.6) is solved be default: the capitalists are unconditionally interested in increasing the α parameter. In employees' case inequation (1.7) is applicable. Its rightmost part grows by *p* and changes its sign at point: $b(p) = \left(\frac{1}{1-p}\right)^{1/p} = 1{,}45524$. At $k \in (1;1{,}45524)$ the employees are interested in lowering α, while at $k > 1{,}45524$ and condition $\alpha < 1$ – in increasing α.

Thus, for the CES-function at $p < 0$ on plane (k,α) there are two areas of social groups' interests concurrence:

a) Within the $\Omega_p^{\downarrow} = \{(k,\alpha): 0 < k < 0{,}68717, \alpha > 0\}$ area, both social groups agree to decrease the α parameter;

b) Within the $\Omega_p^{\uparrow} = \{(k,\alpha): k > 1{,}45524, \alpha < 1\}$ area, both social groups agree to increase the α parameter. Within the remaining area $0 < \alpha < 1$ the social groups' interests diverge.

Table 2

Area construction data $\Omega_p^{\downarrow}$ and $\Omega_p^{\uparrow}$ for the US economy

$\Omega_p^{\downarrow} = \{(k,\alpha): 0 < k < 0{,}68717, \alpha > 0\}$		$\Omega_p^{\uparrow} = \{(k,\alpha): k > 1{,}45524, \alpha < 1\}$	
α	k	α	k
0.1	0.432104	0	1.455244
0.2	0.500321	0.1	1.483218
0.3	0.544125	0.2	1.515721
0.4	0.576808	0.3	1.55415
0.5	0.602959	0.4	1.600611
0.6	0.624762	0.5	1.658487
0.7	0.643439	0.6	1.733679
0.8	0.659752	0.7	1.837811
0.9	0.67421	0.8	1.998716
1.0	0.68717	0.9	2.314258

Based on Table 2 data let's build the interests concurrence/divergence areas for the social groups which are defined as production CES-function's factors. Figures 1, 2 show graphs limiting the social groups' interests concurrence/divergence areas. The most noticeable graphs' shift is for 2009. The social groups' interests divergence area got narrower. The area, where social groups come to an agreement to decrease the α parameter, got bigger. The area, where social groups come to an agreement to increase the α parameter, got bigger as well.

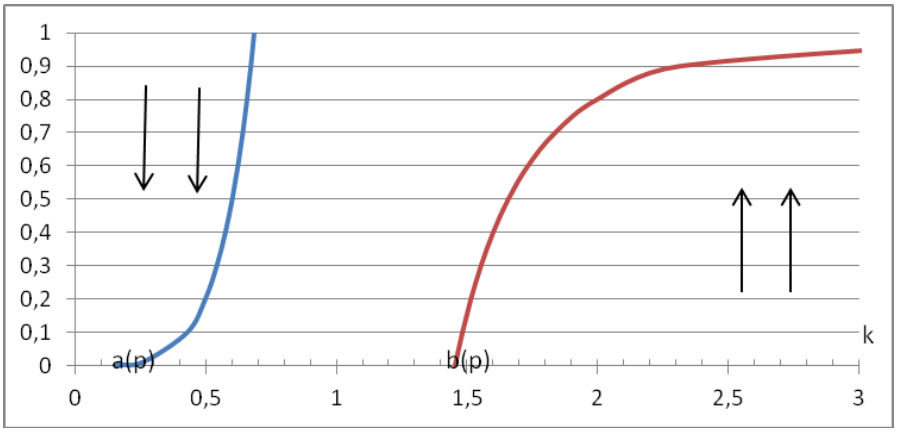

Fig. 1. The social groups' interests concurrence/divergence areas in case of the CES-function for the US economy in 2000

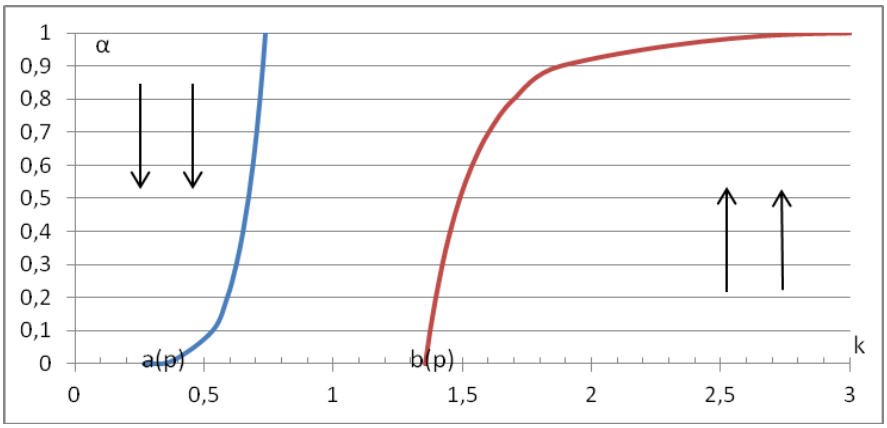

Fig. 2. The social groups' interests concurrence/divergence areas in case of the CES-function for the US economy in 2010

Most distinctive is the behavior of such an integral feature as the size of the social groups' interests divergence area. This integral feature can be calculated for any given year and followed through it evolution. Fig.3 shows this feature's dynamics for 2000-2010. The graph demonstrates that during the years of worsening of the recession the society displays signs of consolidating. The size of the social groups' interests divergence area gets smaller.

Moreover, we can say that the experience of overcoming crises, especially the one gained during the Great Depression, the social-economic institutions established in the country are really effective and the recession in the US results in certain consolidation of social groups' interests.

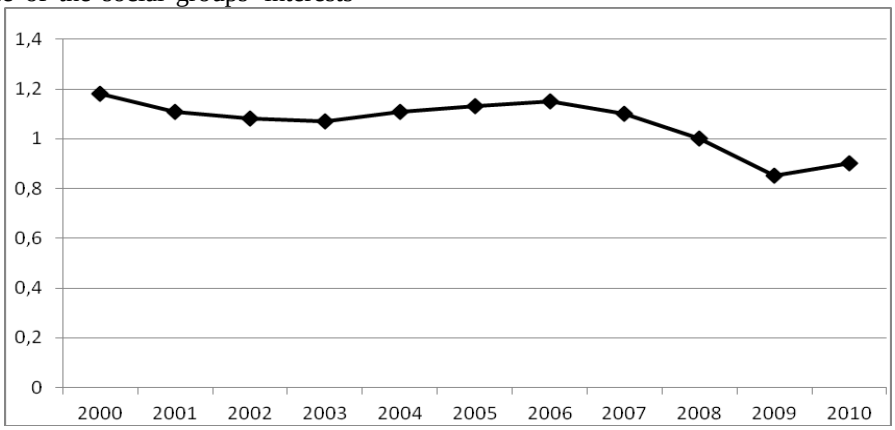

Fig. 3. Fluctuation of the size of the social groups' interests divergence area for the US economy in 2000-2010

Following the above-mentioned method let's analyze social groups interaction dynamics for the economy of Japan. Thus, for the Japanese economy for every given year and the given values $p < 0$ we can obtain the shapes of the social groups' interests concurrence/divergence areas on the plane (k, α)

As in the previous case the distinctive feature is the behavior of an integral feature as the size of the social groups' interests divergence area as seen in Fig. 4.

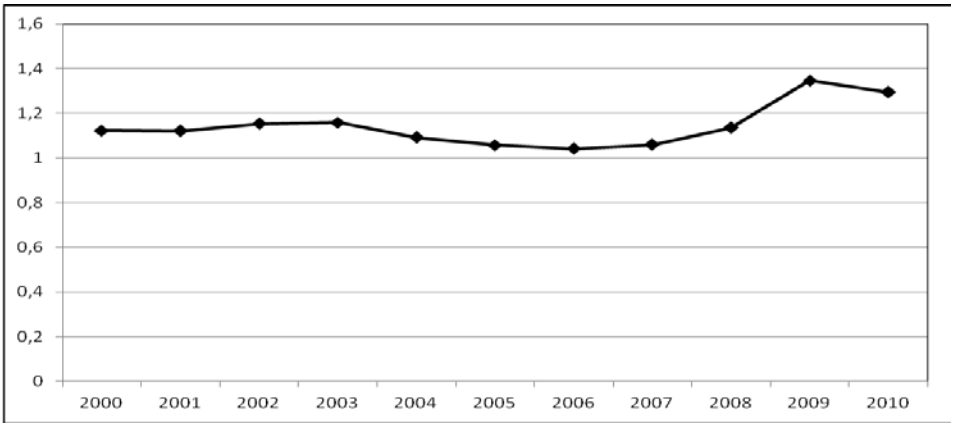

Fig. 4. Fluctuation of the size of the social groups' interests divergence area for the Japanese economy in 2000-2010

This integral feature can be calculated through approximate integration for every year, then the social groups' interaction dynamics in the Japanese economy can be followed.

In Fig.4 one can follow the fluctuations the social groups' interaction in Japan in 2000-2010: within the first 4 years there's the increase of the social groups' interests divergence area. This area corresponds to unfavorable years for the Japanese economy: industrial stagnation, unemployment growth [7]. Then, following the steps taken by the Japanese government, there 4 favorable years. In 2007 Japan's GDP growth rate was 2%. However, in the second half of 2008 following the world economic crisis Japan's economy was plunged into recession. The integral feature graph illustrates this situation peaking up at the year of 2009. It should be noted that the behavior of the integral feature representing social groups' interaction seems to be quite logical. Here is the table of the parameter's value p by years.

Table 3

***P* parameter's value for the Japanese economy in 2000-2010**

Year	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010
p	-3.4977	-3.4715	-3.2297	-3.3079	-3.5210	-3.6730	-3.7387	-3.7846	-3.3775	-2.5521	-2.7423

As it has been demonstrated by the obtained data the increase of the p parameter leads to the similar increase in size of the social groups' interests divergence area, while the opposite is also true, the smaller the p parameter is, the smaller the social groups' interests divergence area gets. Or, considering the bond of p parameter and substitution elasticity σ: the higher factors substitution elasticity is, the greater the social groups' interests divergence area gets and vice versa.

From Fig.4 it's obvious that a greater interests (capitalists vs employees) divergence area covers the year 2009, displaying the parameter's greatest value $p = -2,5521$. At the same time in 2007 (when the parameter's value is at its lowest $p = -3,7846$) the interests divergence area is the least. By the end of 2009 fiscal year Japan was second to the US in the GDP nominal volume, being at that time USD 5 trillion. However, according to some experts' estimates in August 2010 China's economy surpassed Japan's one pushing it back into the third place by the GDP volume and by the purchasing power parity (following the US and China) [1, 7, 12]

3.2 Analysis of the social groups interaction dynamics among developing countries

China, considered a steady capital stasher, is facing now a massive capital outflow from the country. In 2010 almost 225 billion dollars' worth of capital left China, which, according to Forex.ru (2012) [1, 12], makes up almost 3% of the country's GDP. PRC economy's macroeconomic indices are calculated using the economic statistical data [16] as well as method [8]. Using the above-mentioned US-related analysis method, let's analyze the social groups' interaction dynamics in terms of PRC economy's production function factors. The calculations of the interaction integral feature are shown in Fig.5.

The area, where the social groups agree to cut the α parameter, is virtually unchanged, however, the area, where the social groups agree to increase the α parameter, is significantly reduced. Hence, the area, where social groups' interests diverge, starts noticeably growing starting with 2007. This indicates that there were certain problems in PRC economy during the

recession. Even more striking is the fact that the change dynamics of this integral factor is reverse for the US: generally, it's typical of the US economy to lower this factor especially when the recession is at its worst, while the PRC economy demonstrates the factor's rise - the size of the social groups' divergence area grows and is continuing to do so.

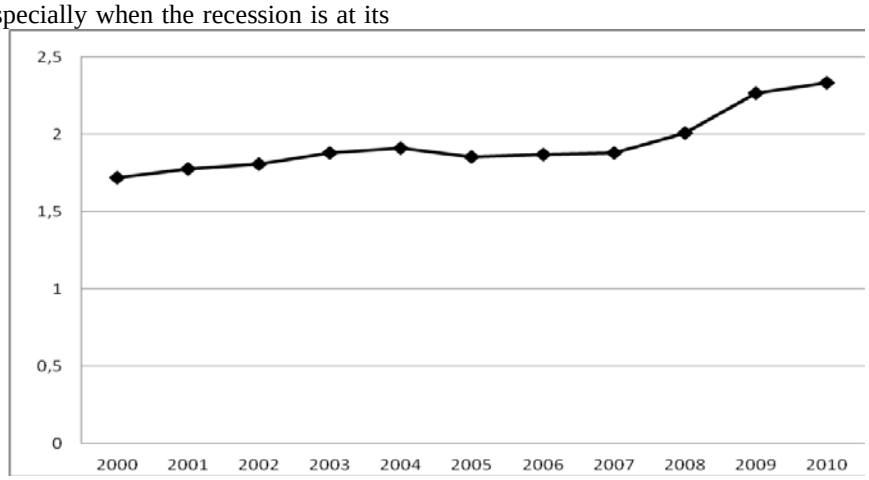

Fig. 5. Changes in PRC social groups' divergence area

Let's analyze the macroeconomic factors dynamics for Russian economy in 2000-2010 using the same method. As it is in China's economy the greatest interests (capitalists vs employees) divergence area covers the year 2010, displaying the parameter's greatest value $p = -0,9238$. At the same time in 2004 (when the parameter's value p is at its lowest of -1.552) the interests divergence area $\Omega_p^{\downarrow\uparrow}$ is the least (Fig. 6).

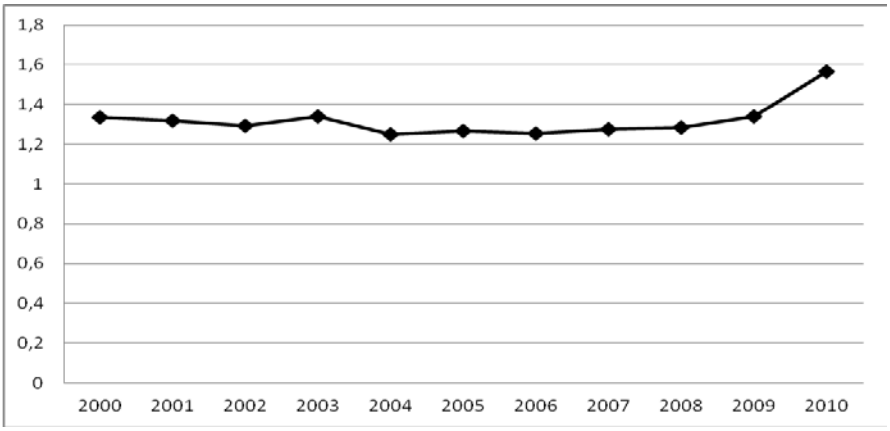

Fig. 6. Changes in Russian social groups' divergence area

Let's draw graphs of changes in social groups' divergence area for Iran (Fig. 7). The obtained graph reflects the fact that Iranian social groups' divergence area differs significantly from those of the USA, China, and Russia, however it's qualitatively closer to Japan. At the same time upon quantitative evaluation of Iran's social groups divergence area we can say that the situation is totally different from that in China (the factor is lower). The difference between the two preceding cases is that on the way out of recession Iranian social groups' divergence area starts shrinking.

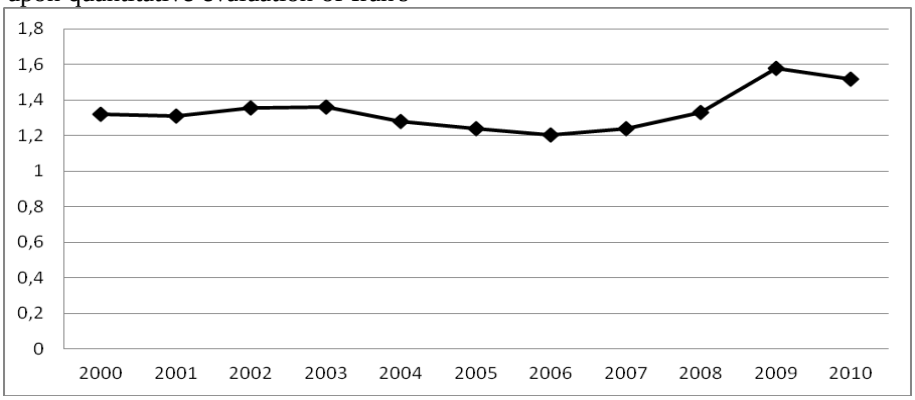

Fig. 7. Changes in social groups' divergence area in Iranian economy

4. Conclusion

The calculations done using the model lead to the preliminary conclusion, that the model in question can identify a crisis and show an economy's reaction to crisis manifestations in terms of social groups interaction. To us more important seems the ability of the model to indirectly statistically evaluate the influence of state institutions on the formation of the social groups while choosing the production function's parameter in particular. To a certain degree the obtained results reflect development trends, including those of major economies - the USA and China. It has been for some time already that in the US social regulation steps were effectively taken, which optimistically promises ease of global recession. In PRC formation of both the regulation instruments and institutions is at its budding phase, and, possibly the prospects of economic development are not as bright as they currently seem.

This model helps follow the evolution curve of a hypothetical developing country. At a low labor-capital ratio a country can find itself straddling the are boundary $\Omega^{\downarrow}p$, later, riding the labor-capital ratio's growth it enter the social-conflict zone, where the income distribution among social groups depends on those groups' negotiations power as well as ather institutions. In many instances one can expect the labor's share to increase. At a further increase of the labor-capital ratio, the country enters area $\Omega^{\uparrow}p$, at which point the both groups agree to increase the capital share, meaning that often the employees themselves become stockholders. It seems that this very factor is decisive in the groups interactions for the US economy.

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МОДЕЛИРОВАНИЕ ЭКОНОМИЧЕСКОГО РОСТА РАЗЛИЧНЫХ СТРАН С ПОМОЩЬЮ ПРОИЗВОДСТВЕННЫХ ФУНКЦИЙ НА ОСНОВЕ СРАВНИТЕЛЬНОГО АНАЛИЗА ДИНАМИКИ ВЗАИМОДЕЙСТВИЯ СОЦИАЛЬНЫХ ГРУПП

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На основании теории эндогенного роста мы даем возможное объяснение различий в развитии народных хозяйств мира и их реакции на глобальные экономические кризисы. Ранее одним из авторов (Матвеенко, 2007, 2010) был предложен подход к моделированию выбора технологий в странах. Технический прогресс смоделирован как изменение параметров производственной функции CES $Y = A(\alpha K^{\rho} + (1-\alpha)L^{\rho})^{1/\rho}$. Параметры определяют доли факторов производственной функции, в частности, для функции Кобба – Дугласа это доля капитала α. Большое различие α в разных странах было найдено недавно опытным путем несколькими авторами. Выбор параметра зависит от интересов социальных групп: работники и владельцы капиталов соглашаются на изменение параметра α, если это приводит к увеличению дохода этой группы. Таким образом, можно определить области совпадения и несовпадения интересов социальных групп в плоскости α–k, где k – отношение капитала к труду. При помощи статистических данных UNSTATS мы строим графики параметрических зависимостей α–k для экономических систем США, Японии, России, Китая и Ирана в течение периода 2000–2010 гг. Области совпадения/несовпадения интересов и их изменения на графиках отражают подпериоды экономических кризисов.

Ключевые слова: моделирование, экономический рост, производственные функции, динамика взаимодействия социальных групп.

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